SEEMOUS 2020

Thessaloniki, Greece
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Problem 1. Consider $A \in \mathcal{M}_{2020}(\mathbb{C})$ such that

$$(1) \qquad \begin{aligned} A + A^{\times} &= I_{2020}, \\ A \cdot A^{\times} &= I_{2020}, \end{aligned}$$

where $A^{\times}$ is the adjugate matrix of A, i.e., the matrix whose elements are $a_{ij} = (-1)^{i+j} d_{ji}$, where d_{ji} is the determinant obtained from A, eliminating the line j and the column i.

Find the maximum number of matrices verifying (1) such that any two of them are not similar.

Problem 2. Let $k > 1$ be a real number. Calculate:

(a) $L = \lim\limits_{n\to\infty} \displaystyle\int_0^1 \left(\frac{k}{\sqrt[n]{x} + k - 1} \right)^n \mathrm{d}x.$

(b) $\lim\limits_{n\to\infty} n \left[L - \displaystyle\int_0^1 \left(\frac{k}{\sqrt[n]{x} + k - 1} \right)^n \mathrm{d}x \right].$

Problem 3. Let n be a positive integer, $k \in \mathbb{C}$ and $A \in \mathcal{M}_n(\mathbb{C})$ such that $\operatorname{Tr} A \neq 0$ and

$$\operatorname{rank} A + \operatorname{rank} \left((\operatorname{Tr} A) \cdot I_n - kA \right) = n.$$

Find rank A.

Problem 4. Consider $0 < a < T$, $D = \mathbb{R} \setminus \{kT + a \mid k \in \mathbb{Z}\}$, and let $f : D \to \mathbb{R}$ a $T-$periodic and differentiable function which satisfies $f' > 1$ on $(0, a)$ and

$$f(0) = 0, \quad \lim_{\substack{x\to a \\ x<a}} f(x) = +\infty \text{ and } \lim_{\substack{x\to a \\ x<a}} \frac{f'(x)}{f^2(x)} = 1.$$

(a) Prove that for every $n \in \mathbb{N}^*$, the equation $f(x) = x$ has a unique solution in the interval $(nT, nT + a)$, denoted x_n.

(b) Let $y_n = nT + a - x_n$ and $z_n = \displaystyle\int_0^{y_n} f(x)dx$. Prove that $\lim\limits_{n\to\infty} y_n = 0$ and study the convergence of the series $\sum_{n=1}^{\infty} y_n$ and $\sum_{n=1}^{\infty} z_n$.